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INSIDER TRADING, ABNORMAL RETURN AND PREFERENTIAL INFORMATION: SUPERVISING THROUGH A PROBABILISTIC MODEL

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Insider Trading, Abnormal Return and Preferential Information: Supervising through a Probabilistic Model

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Abstract The enforcement of the ban on insider trading requires an evaluation of the disgorgement, i.e. the capital gain of the insider trader who takes advantage of the exploitation of preferential information. An initial step forward on this topic has been taken by the SEC, the United States Securities and Exchange Commission, which has developed a quantitative procedure based on the event-study methodology. This paper, as first approximation develops an adaptation of this procedure for the Italian market in order to better explain the limits of this methodology in the analysis of the insider-trading phenomenon. In particular, it emerges that the econometric approach cannot be applied to all insider-trading schemes. In fact, in order to work out statistically significant results, it relies on a series of assumptions such as the existence of a robust reference market index or the availability of long time series data. For this reason, a new procedure for computing the economic value of the information exploited by the insider, based on a probabilistic approach, has also been developed. This methodology overcomes the issues connected to the event-study procedure and can be applied by construction to all insider-trading schemes and not only to the simplest ones. In fact, the model parameters are defined by using the trading strategy of the single insider; thus, if insider trading takes place, the model is able to offer a disgorgement computation; hence, by hypotheses of its construction, it is able to detect the difference between insiders and followers. The new procedure has been adopted by CONSOB and have been presented to the Judicial System.

Key words: SEC, CONSOB, insider trading, disgorgement, event studies analysis, proxy variable, geometric Brownian motion, wiener process, abnormal return, cumulative abnormal return.

[1] CONSOB, Intermediaries Division - Enforcement Officer.
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1 Introduction

On a world-wide scale, of a total of 103 countries that have stock markets, 87 of them have regulated the insider-trading phenomenon (Bhattacharya, Daouk, 1999). This situation is the result of a dispute between the two main theoretical streams, which can be succinctly represented as follows: the first is convinced that a ban on insider trading reduces market efficiency and managers' compensation, while the other states that the insider trader appropriates the value of the preferential information[2] to the detriment of other investors and consequently the repression of this crime increases the investors' trust in the market, and hence its liquidity. These theoretical streams have developed their arguments in more than 250 papers over the last forty years. These arguments can be summarized in three theories in favor of the repression of insider trading and three others against it (Bainbridge, 1988).

The three theories against the enforcement of a ban on insider trading can be defined as follows:

1. victimless crime;
2. managers' compensation;
3. market efficiency.

The first one (Herzel and Katz, 1987[3]) states that insider trading has no victim; this is because transactions carried out by the insider moves the stock price in the same direction as preferential information and consequently the counterpart of the insider also takes advantage of the insider's transactions. For instance, in the case of bullish information the insider would raise the stock price and consequently the counterpart would sell the stock at a higher price than he would have without the insider transactions.

The second theory is based on the concept that the only effective way to compensate managers is through the exploitation of preferential information. This is because of the fact that bonus and stock options are not flexible enough and financially viable for the company (Manne, 1966).

The latest theory against the regulation of the crime exploits the concept of market efficiency in its strong form, i.e. the stock price reflects all available information, preferential included. Hence, by carrying out his strategy, the insider pushes the stock price faster towards the value which better reflects the fundamentals of the company (Finnerty, 1976).

The three theories which support the repression of insider trading can be defined as follows:

1. misappropriation theory;
2. market egalitarianism;
3. market integrity.

The first theory bases its main argument on the idea that preferential information is property of the company. Therefore, any exploitation of information

[2] It is reported in Appendix A a glossary of the terms used in the paper.

[3] The view that insider trading is a "victimless crime" is a popular one. Hertzel and Katz, in their paper explain this theory and criticize it.

carried out by a subject other than the owner, i.e. the company, could be assimilated to theft (Georges 1976).

The market egalitarianism theory is based upon the argument that all investors should make their investment decisions on the basis of the same information set, in order to have the same pay-off opportunities (Loss 1983 Langevoort 1987).

Finally, the third theory emphasizes the concept of market integrity. This theory argues that the insider trader damages the market, particularly its microstructure. This damage moves through two main channels: the first operates as a chain reaction involving the operativity of the market makers and the investors' trading; the second concerns the investment decisions of the institutional investors. As far as the first channel is concerned, the presence of insider traders in the market creates losses to the market makers who, in order to maintain long-term profitability, tend to increase the bid-ask spreads. This situation creates an increase in transaction costs, which operates as a tax on all investors, and creates a disincentive in trading activity. These effects cause, firstly, a decrease in the liquidity of the market and in the signalling role played by price, secondly a reduction of market efficiency and lastly an increase in the cost of capital for the companies[4] (King and Roell, 1988; Bhattacharya, Daouk, 1999). The main assumption behind the damage to the integrity of the market with reference to the second channel is that the institutional investors bear high research costs in order to define their investment decisions, and consequently their trading follows the results of the analyses of the company fundamentals. Unfortunately, the insiders' trading influences the stock prices according to a dynamic determined by the value of the preferential information instead of the company fundamentals. Therefore, the stock prices will diverge from the prices which reflect the companies fundamentals. This trend represents a misleading indication for institutional investors so that they trade against this dynamic. It implies that the institutional investors' trading will lose significantly according to the value of the inside information. This loss plays a disincentive role for this category of operators to invest in research and consequently it activates a vicious circle that lead the market prices far from the fundamentals values, undermining the market integrity (Milia, 2000)

This brief analysis of these theories and of their different arguments offers an easy explanation of why only some of the countries with a stock market have regulated insider trading. Unfortunately, a more in-depth analysis of the phenomenon shows, that out of 87 countries, only 38 have really enforced this crime (Bhattacharya, Daouk, 1999).

This consideration highlights a new worrying perspective on this subject that cannot be restricted to the aforementioned theoretical dispute. In fact, the enforcement on the ban on insider trading presents several operative issues for the supervisors.

Some quantitative procedures have to be used in order to detect the phenomenon, to compute the value of preferential information and hence, to calcu-

[4] This is also the main argument of the Regulators.

late the disgorgement, which is the undue wealth gained by the insider through the exploitation of preferential information.

While the detection phase of the insider affects the level of sensitivity in the market analysis carried out by the supervisor, and hence the amount of signals that have to be examined, the evaluation of the disgorgement offers, in all legal systems punishing the crime of insider trading, a benchmark to identify the sanction to be imposed against the insider. As a result, it can be considered as the link between the financial and legal aspects.

Therefore in the enforcement of the ban on this crime, the supervisors have to be accurate in the identification of the value of the information that the insider trader would appropriate (Mitchell, Netter, 1994).

Hence, the difficulty in identifying an objective, realistic, and effective way of computing this value can give raise to problems in assessing the damage caused by the insider to the market and, consequently, to the enforcement action.

One of the most important contributions to this subject has been made by the SEC, which in the 1980s developed the first quantitative methodology for computing the disgorgement by developing an econometric approach based on the event-studies theory.

The purpose of this paper is fivefold. First, it sketches the legal references for the prosecution of insider trading in the United States of America and in Europe with a breakdown of how Italy, France, Germany and the United Kingdom have adopted the European directive. This breakdown will highlight the fact that it is common practice to use the disgorgement gained by the insider as the main reference for the determination of the sanctions. Secondly, it illustrates the methodologies used by the regulators, their respective limits and why the SEC has developed an econometric procedure. Thirdly, it wholly explains the rationale behind this procedure and how it develops operatively. To this end it illustrates an adaptation of this procedure to the Italian market and how it is used by regulators. Fourthly, it explains the limits of the econometric procedures and why they are not applicable to all insider-trading schemes. Hence it presents a new methodology for studying the insider-trading phenomenon which, unlike the event-study theory, adopts a probabilistic approach. The paper shows the advantages of this procedure and why from a methodological point of view it is theoretically superior to previous procedures; it also demontrates its applicability to all insider-trading investigation cases and its ability to distinguish between insiders and followers.

The new procedure has been adopted by CONSOB and it has also been presented to the Judicial System, which deals with most of the insider-trading cases.

2 The legal framework for the repression of insider trading

This section considers the prosecution of the crime of insider trading, according to the different legal frameworks of various countries. It illustrates the scope of the rules, the definition of preferential information, the subjects under supervision and what type of behavior is forbidden.

In general terms, three main rules have to be taken into account in the repression of the crime:

1. home-country control: every country is in charge of the monitoring of insider trading on the stocks quoted in the stock exchange established in its territory;

2. co-operation between the authority in charge of the insider-trading control and the judicial system;

3. disgorgement computation in order to quantify the damage that the insiders have inflicted on the market as a result of their trading.

2.1 The legal establishment in the USA

In the United States the first prosecution of insider trading under State law occurred in 1903. In spite of this fact, the legal establishment for the repression of insider trading was only with Section 16 of the Securities Exchange Act in 1934. This law did not take into account the use of preferential information made by insiders and imposed the prohibitions only on directors, officers and those shareholders having more than 10% of the registered capital. Due to these objective and subjective limitations, the SEC adopted the proxi considered in Section 10 (b) of the above-mentioned act in order to enact the rules protecting the stock exchange from fraud. As a result, the SEC drew up the rule 10b-5 in 1942. This rule, following Section 17 of the Securities Act of 1933, removed the subjective limitations set out in Section 16 and eliminated a clear loophole in the law, introducing the case of the acquisition of securities, which had not been included in Section 17 of the Securities Act of 1933 (Loss 1970). Since 1942, the rules adopted to punish the crime of insider trading in the United States and the discipline as a whole have been affected by several interpretations given by SEC and the law. At the end of the 1970s, the Second Circuit Court of Appeal pointed out an evident limitation of this discipline: the continuous series of laws enacted between 1942 and 1980 required a fiduciary duty between the seller and the counterpart in order to contemplate the crime of insider trading. As huge numbers of take-overs, mergers and acquisitions, took place in the USA, at the end of the 1970s, this law proved to be inadequate. On October 14th 1980, the SEC, empowered by Section 14-e of the Securities Exchange Act of 1934, enacted rule 14e to remove this additional subjective restriction.

The rules mentioned above, coupled with several decisions made by the District of Columbia, the Second and Ninth Circuit Courts of first instance and the Second, Fourth, Eighth Circuit Courts of Appeal and the Supreme Court, represent the legal framework of reference in the United States[5].

The basic elements of the discipline are the following:

Scope

The prohibition is imposed on the purchasing and selling on those securities listed on one of the national Stock Exchanges and carried out on the market or in transactions carried out by individuals without financial intermediaries and outside the regulated markets (i.e. face-to-face transactions).

Preferential information

The prohibition is imposed on material and non-public information. In compliance with what the Supreme Court has set out, a piece of information has to be considered as material when a "reasonable" investor sees its disclosure as of paramount importance for an investment. When corporate (information related to the issuing body) or market information (information related to the whole market or to the sector in which the issuing body works) is kept secret it is considered as non public.

Prohibitions are imposed on the following subjects

A) every subject with knowledge of preferential information;

B) every subject having fiduciary duty towards the owner of the information (misappropriation theory);

C) every subject that receives information that is non public ("tippee")

Forbidden behavior

Insiders are not allowed:

- to carry out financial transactions when they consciously have preferential information (prohibition of trading);

- to provide third parties with this information (tipping);

- to suggest that a third party should carry out transactions in the market based on this information (tuyautage);

- to prompt a third party to carry out transactions.

As far as the role of the SEC and of the judicial system in the repression of this crime in the USA are concerned, the SEC is empowered to undertake civil actions and interacts with the judicial system for penal actions. Both actions provide for the application of fines, which are computed in relation to the disgorgement.

2.2 The legal establishment in Europe

In Europe the regulation of insider trading is covered by the EEC Directive 89/592 (November 13th, 1989); its basic elements are the following:

Scope

Article 1 states that the law can only be enforced on financial transactions carried out on a market which is "regulated and supervised by authorities recognized by public bodies" and which "operates regularly and is accessible directly or indirectly to the public".

[5] A good recapitulation of these decisions can be found in Georges (1976), Loss (1983), Langevoort (1987), Hagen (1988), Martin (1986), Kraakman (1991) and Bergmans (1991).

Paragraph 3 of Article 2 states that the law has to be enforced only on those transactions taking place with the intervention of a professional intermediary. Each member state is empowered to enforce the law on those transactions carried out by individuals without financial intermediaries and outside the regulated markets (face-to-face transactions).

Article 5 defines the territorial jurisdiction: each member state is entitled to enforce the prohibitions "at least to actions undertaken within its territory to the extent that the transferable securities concerned are admitted to trading on a market of a Member State." In any case, each member state has to take in those transactions related to real values carried out inside a regulated market "situated or operating within that territory."

Preferential information

Article 1, n.1 of the directive defines inside information as "information which has not been made public of a precise nature relating to one or several issuers of transferable securities or to one of several transferable securities, which, if it were made public, would be likely to have a significant effect on the price of the transferable security or securities in question."

Hence, preferential information consists of specific corporate information related to the issuing body (corporate information) or of general information related to the whole market or to the sector in which the issuing body works (market information).

Moreover, preferential information is to be kept secret. In compliance with the current law, this kind of information is no longer considered to be preferential when it is accessible to other parties even though they do not actually know of it.

Prohibitions are imposed on the following subjects

A) institutional insiders: those "by virtue of [their] membership of the administrative, management or supervisory bodies of the issuer" who have preferential information (Art. 2, n.1);

B) other primary insiders: those that have access to the information "by virtue of the exercise of their employment, profession or duties" (Art. 2, n.1);

C) "tippee": as for Article 4, he is "any person other than those referred to" in the Art. 2 (basic or institutional insiders) who "with full knowledge of the facts possesses inside information, the direct or indirect source of which could not be other than a person referred to in Art.2".

Forbidden behavior

The subjects mentioned in A and B are not allowed:

- to buy or sell, on their account or on behalf of a third party, directly or indirectly, those real values related to the preferential information (prohibition of trading) deliberately using the information;

- to provide third parties with preferential information "unless such disclosure is made in the normal course of the exercise of his employment, profession or duties" (tipping);

- to suggest that a third party should carry out transactions related to the real values that the preferential information is about (tuyautage);

- to prompt a third party to carry out transactions.

The subject mentioned in C is not allowed to:

- trade, even though each member state can also impose the prohibition of tipping and tuyautage on this subject, which is usually imposed on institutional or basic insiders.

As far as the role of the authority empowered with insider-trading control and of the judicial system in the repression of the crime in the EEC is concerned, the following table gives a comprehensive explanation of how Italy, France, UK and Germany have adopted the European Directive and highlights the importance of the disgorgement computation for the definition of the fine.

	Italy	Germany	United Kingdom		France	
Law Reference	Consolidated Act on Financial Intermediation (1998)	Securities Trading Act (1994)	Criminal Justice act (1993)	Financial Service and Market Act (2000) - FSA code (2000)	COB Rule 90-08 (1991)	Ordonnance 67-833 (1967)
Scope	Any Securities listed on a Domestic or EEC stock exchange Also Face -to - Face Transactions	Any Securities listed on a Domestic or EEC stock exchange Also Face -to - Face Transactions	Any Securities listed on a Domestic or EEC stock exchange	A qualifying investment on a prescribed market or accessible electronically in the UK	Any Securities listed on a Domestic stock exchange Also Face -to - Face Transactions	Any Securities listed on a Domestic stock exchange Also Face -to - Face Transactions
Inside Information	non Public, Precise and Significantly Price Sensitive	non Public and Price Sensitive	non Public, Precise and Significantly Price Sensitive	Non generally Available and relevant to a regular user	non Public, Precise and Price Sensitive	non Public and Price Sensitive
Subjects Enforced	institutional insiders Other Primary Insiders Tippees	institutional insiders Other Primary Insiders Tippees*	institutional insiders Other Primary Insiders Tippees	institutional insiders Other Primary Insiders Tippees	institutional insiders Other Primary Insiders Tippees*	institutional insiders Other Primary Insiders
Forbidden Behaviour	Trading -on own account -On behalf of third parties Suggest the preferential information to third parties Communicate the Information to third parties	Trading* -on own account -On behalf of third parties Encourage third parties to exploit the preferential information Communicate the Information to third parties	Trading -on own account -On behalf of third parties Encourage third parties to exploit the preferential information Communicate the Information to third parties	Trading -on own account -On behalf of third parties Encourage third parties to exploit the preferential information Communicate the Information to third parties	Trading* -on own account -On behalf of third parties Encourage third parties to exploit the preferential information Communicate the Information to third parties	Trading -on own account -On behalf of third parties Encourage third parties to exploit the preferential information Communicate the Information to third parties
Sanctions Penal Sanctions Subjects in Charge of Carrying out the Sanctions	Imprisonment Fine: (also) in relation to the Disgorgement Judicial System	Imprisonment Fine: in relation to the Disgorgement Judicial System	Imprisonment Fine: no limit (also in relation to the Disgorgment) Judicial System			Imprisonment Fine: in relation to the Disgorgement Judicial System
Civil Sanctions Subjects in Charge of Carrying out the Sanctions			Fine: in relation to the Disgorgement Inprisonment* FSA, LSE, DTI, Bank of England, BSC, CPS*			
Administrative Sanctions Subjects in Charge of Carrying out the Sanctions		Suspend the Authorization of providing Financial Services. BAEW		Fine: (also) in relation to the Disgorgement FSA	Fine: in relation to the Disgorgement COB, CBV, CMT	

BAEW: The Germany Securities and Exchange Commission

LSE: The London Stock Exchange
DTI: Department of Trade nad Industry
BSC: The Building Society Commission
CPS: The Crown Prosecution Service

FSA: The Financial Service Authority

COB: The French Securities and Exchange Commission

CBV, CMT: the French Security Exchange

3 Computation of the disgorgement

As already stated, the laws prohibiting insider trading identify in the disgorgement a benchmark for quantifying the sanctions against the insider[6]. Therefore careful evaluation is necessary and has to be applied to all cases provided for and accepted by the judicial power in those countries where the legal system empowers them to impose similar sanctions.

The first method of evaluation, adopted by the supervisors, consists in calculating the actual disgorgement. In this case, the computation corresponds to the difference between the value of the insiders' closed position over the security (usually after the disclosure of the preferential information) and the value of his open position. However, this method is not effective if the insider closes the position well after the disclosure of the information or if the position is not closed at all; in this case, the connection between the information and the insider trading may vanish.

In order to overcome these difficulties, the supervisors generally calculate the disgorgement as the difference between the price after the disclosure of the information and the weighted average price of the insider open position, multiplied by the invested quantities. This methodology is defined as *potential deterministic disgorgement.*

Yet, this procedure can also give raise to some problems; for example, if the insider opens the position well before the disclosure of the information, its profitability may be affected by events unrelated to its trading.

In order to tackle all these problems, the SEC has developed a procedure on the basis of the Event Study Analysis that allows the determination of the return percentage variation of the security caused by the preferential information. This computation is based on the relationship between the return obtained by the security and the return of the reference market index. This is defined *potential econometric disgorgement.*

The potential-econometric-disgorgement method has improved the procedure regarding the evaluation of the profit gained by the potential insiders; an adjustment of this procedure to the Italian market has been developped. However, this method causes some difficulties and therefore cannot be applied to all insider-trading investigation cases, such as the individuation of a statistically robust market-proxy portfolio, the need for a long historical time series data set and the condition that a linear deterministic relation found in the past is also stable and effective in the future.

To overcome these issues a new methodology to study the insider-trading phenomena based on a probabilistic approach has been developed and it is currently used within CONSOB. The procedure leads to the computation of the *potential probabilistic disgorgement* by analyzing all the future price scenarios, assigning them a suitable probability measure, on the basis of the strategy of the insider and on the current stock price.

[6] As shown in the previous paragraph, it is important to highlight, that the importance of the disgorgement and its role in the determination of the sanctions against the insider is not the same in the different legislations.

In order to have a clearer understanding of how the potential econometric disgorgement is operatively calculated both in the American and Italian methodologies, and of the problems related to this computation, which have led to a new probabilistic approach, it is necessary to analyze how and why the Event Study Analysis has traditionally been developed.

3.1 Potential Econometric Disgorgement

3.1.1 Event Study: the traditional approach

The evaluation of the impact of an event on the value of a company is a difficult task for economists. The event-study analysis, which estimates the effect on stock returns of occurrences, such as mergers, acquisitions, takeovers, announcements, variation of the regulation in the reference microeconomical system, etc., is widely used. The first publication concerning the event-study methodology dates back to 1933 (Dolley). Over the years this methodology has been applied in a variety of fields, such as the study of insider-trading phenomena[7].

The traditional methodology consists of nine fundamental steps:

1. the definition of the events to be studied and the reference time horizons for the analysis.

Supposing that the date of the event is Γ, there is a time horizon used for the estimate of the model parameter $\alpha = T_0 \rightarrow T_1$ that is defined before the event, a time horizon which contains the event $\Theta = T_{1+1} \rightarrow T_2$ for a verification of the significance of the regression model defined in the period α and consequently for the estimate of the effect on stock return of the event just highlighted (figure 1);

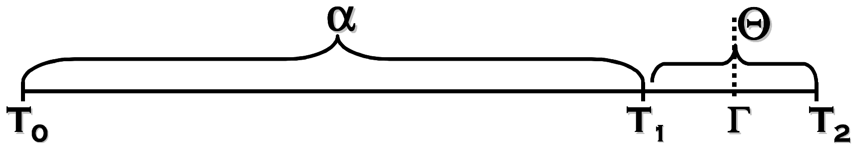

Figure 1:

2. analysis of the company history in the reference time horizon in order to detect the variations of the company stocks value; to avoid the presence of breaks in the series of stock returns due to information heterogeneity.

Returns[8] are defined as:

$\ln \frac{S_t}{S_{t-1}}$ where S_t is the value of the stock in time t.

[7] For an exaustive discussion of the increasing level of sophistication of the event study over the decades, see Copeland and Weston, (1992), Myers and Bakay (1948), Fama, Fisher, Jensen and Roll (1969).

[8] The dividends can be included or not in the analysis, simply by fitting the definition of the stock return.

This is done because it is hypothesized that the return $\frac{S_t}{S_{t-1}}$ has a lognormal distribution, therefore the logarithm of this random variable is distributed as a normal:

$\ln \frac{S_t}{S_{t-1}} \sim N(\mu - \frac{\sigma^2}{2}, \sigma^2)$

3. determination of parameters to be employed for the assessment of the normal and, as a consequence, of the abnormal return. A widely employed statistical model is the Market model which explains the relationship between the returns of the ith firm and the market portfolio through the linear regression model:

$R_{it} = \beta_{0i} + \beta_{1i} R_{mt} + \epsilon_{it}$

that is:

$E(R_{it}) = \beta_{0i} + \beta_{1i} E(R_{mt})$

where the model hypothesizes that

i. the regressor observations are independent among themselves;

ii. $\epsilon_i \sim N(0, V_i)$;

iii. ϵ_i are independent random variables; this means that there is not a serial correlation between the errors:

$Cov(\epsilon_{it}, \epsilon_{i\tau}) = 0 \qquad \text{where } t \neq \tau.$

In graphic terms the simple linear regression model can be represented as shown in figure 2:

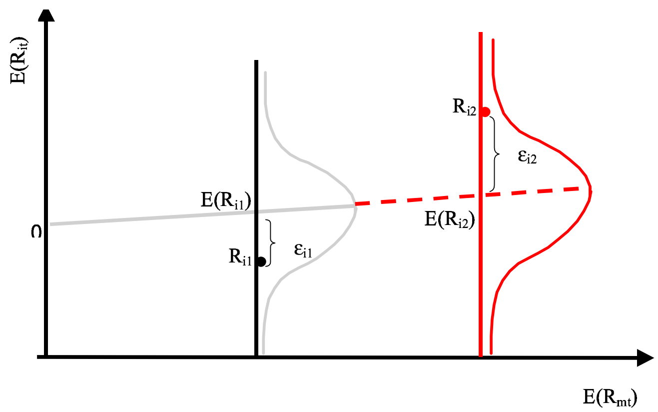

Figure 2:

Generally speaking, in the whole time horizon α the model can be rewritten for the ith stock as follows:

$$\begin{bmatrix} R_{iT_0+1} \\ R_{iT_0+2} \\ . \\ . \\ R_{iT_1} \end{bmatrix} = \begin{bmatrix} 1 & R_{mT_0+1} \\ 1 & R_{mT_0+2} \\ . & . \\ . & . \\ 1 & R_{mT_1} \end{bmatrix} \cdot \begin{bmatrix} \beta_{0i} \\ \beta_{1i} \end{bmatrix} + \begin{bmatrix} \epsilon_{iT_0+1} \\ \epsilon_{iT_0+2} \\ . \\ . \\ \epsilon_{iT_1} \end{bmatrix}$$

that is

$\underset{\alpha x1}{\mathbf{R}_i} = \underset{\alpha x2}{\mathbf{R}_m} \underset{2x1}{\beta} + \underset{\alpha x1}{\epsilon_i}$

4. obviously, the estimate of parameters takes place for every ith stock with the Ordinary Least Squared (OLS) method in the period that is:

$\min_{\beta} \sum_{t \in \alpha} (R_{it} - E(R_{it}))^2$

that in matrix notation equals:

$\min_{\beta} (\underset{\alpha x1}{\mathbf{R}_i} - \underset{\alpha x2}{\mathbf{R}_m} \underset{2x1}{\beta})' \cdot (\underset{\alpha x1}{\mathbf{R}_i} - \underset{\alpha x2}{\mathbf{R}_m} \underset{2x1}{\beta})$

The result of this minimization leads to the identification of estimators $\widehat{\beta_{0i}}$, $\widehat{\beta_{1i}}$ for parameters β_{0i} and β_{1i}.

$$\begin{bmatrix} \widehat{\beta_{0i}} \\ \widehat{\beta_{1i}} \end{bmatrix} = \left\{ \begin{bmatrix} 1 & 1 & . & . & 1 \\ R_{mT_{0+1}} & R_{mT_{0+2}} & . & . & R_{mT_1} \end{bmatrix} \begin{bmatrix} 1 & R_{mT_{0+1}} \\ 1 & R_{mT_{0+2}} \\ . & . \\ . & . \\ 1 & R_{mT_1} \end{bmatrix} \right\}^{-1} \cdot$$

$$\cdot \begin{bmatrix} 1 & 1 & . & . & 1 \\ R_{mT_{0+1}} & R_{mT_{0+2}} & . & . & R_{mT_1} \end{bmatrix} \cdot \begin{bmatrix} R_{iT_{0+1}} \\ R_{iT_{0+2}} \\ . \\ . \\ R_{iT_1} \end{bmatrix}$$

where the estimator $\underset{2x1}{\widehat{\beta}}$ is consistent by construction: $E(\underset{2x1}{\widehat{\beta}}) = \underset{2x1}{\beta}$. The identification of these parameters is necessary for the definition of the regression line for the single ith stock where the ithreturn is generally called *fitted* and the market return *regressor*, as shown in figure 3.[9]

5. these parameters $\overrightarrow{\widehat{\beta}}$ are therefore employed in the time horizon Θ as indicators of the normal return and so as a statistical basis to identify the abnormal return. In fact, employing the regression model:

$\underset{\Theta x1}{\mathbf{R}_i} = \underset{\Theta x2}{\mathbf{R}_m} \underset{2x1}{\widehat{\beta}} + \underset{\Theta x1}{\mathbf{e}_i}$

thus the estimation error $\overrightarrow{\mathbf{e}_i}$ represents the error in the normal return estimation given by the regression model. This error is defined as *potential abnormal return* (AR) and can be identified as the estimation error in the prediction during the period Θ based on regression parameters determined in the horizon α:

$\overrightarrow{\mathbf{e}_i} = \overrightarrow{\mathbf{AR}_i} = \overrightarrow{\mathbf{R}_i} - \mathbf{R}_m \overrightarrow{\widehat{\beta}}$

For the hypotheses of the model the potential abnormal return distribution is normal with the following parameters:

$\overrightarrow{\mathbf{AR}_i} \sim N(0, V_i)$[10]

where

[9] From now on the vector notations $\underset{nx1}{X}$ and $\overrightarrow{X}$ and the matrix notations $\underset{nxm}{X}$ and $\mathbf{X}$ will be used indifferently.

[10] The demonstration of the values assumed by the mean and the variance of r.v. AR is shown in Appendix B.

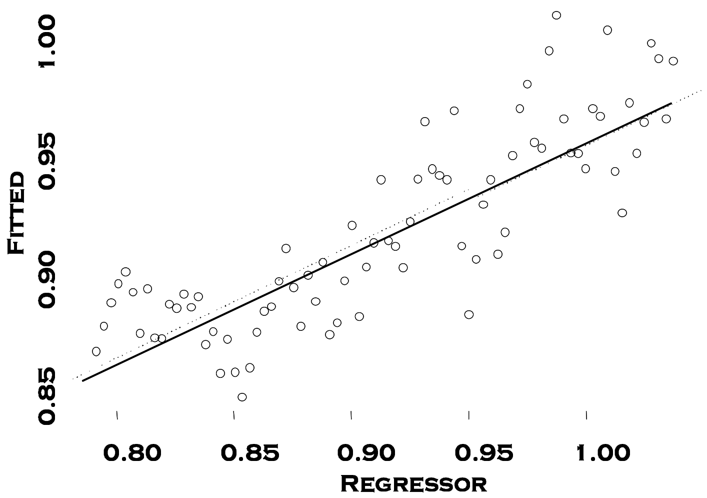

Figure 3:

$V_i = I \cdot \sigma_i^2 + E(\mathbf{R}_m(\widehat{\beta} - \beta)(\widehat{\beta} - \beta)^{'}\mathbf{R}_m' \mid \mathbf{R}_m)$

The second term shows a dependence on the market returns vector, breaking, in this way, the hypothesis of independence of the regressor observations. It is important to note that as the length of period α increases, this serial correlation vanishes because when this estimation interval increases, the term $(\widehat{\beta} - \beta)$ is frustrated.

6. construction of a statistic in order to verify more easily the level of abnormality expressed by the r.v. AR in the period Θ, compared to the model built in the period α. By using, the statistical distribution of the i^{th} AR, and by defining $\overrightarrow{S_i}$ as the vector of the standard deviation of the $\overrightarrow{AR}$, i.e.[11]:

$\overrightarrow{S_i}(\Theta) = \begin{pmatrix} \sigma_{i_{T_{1+1}}} & \sigma_{i_{T_{1+2}}} & \cdots & \sigma_{i_{T_2-1}} & \sigma_{i_{T_2}} \end{pmatrix}$

it is possible to define the r.v. SAR, *Standardized Potential Abnormal Return*, which will be distributed by construction as follows:

$\overrightarrow{\mathbf{SAR}_i}(\Theta) = \frac{\overrightarrow{\mathbf{AR}_i}(\Theta)}{\overrightarrow{S_i}} \sim N(0,1)$

In order to define the distribution of the SAR, ignorance of the value σ_i calls for the employment of an estimator. The estimator to be employed is simply the standard deviation estimator connected with the *fitted* prediction in the period Θ, employing the parameters determined in the period α. The single element of the vector $\overrightarrow{\widehat{S}_i}$ is determined as follows:

$\widehat{\sigma}_{i_{T_1+1}} = \sqrt{\frac{\sum_{\alpha} e_i^2}{n-2}\left(1 + \frac{1}{N} + \frac{\left(\overline{R_m} - R_{m_{T_1+1}}\right)^2}{\sum_{\alpha}\left(R_{tm} - \overline{R_m}\right)^2}\right)}$

[11] The underlying hypothesis is that the second term of V_i vanishes.

By defining $\overrightarrow{\widehat{S_i'}}$ the vector of the estimated standard deviation of the ith r.v. AR, i.e.:

$\overrightarrow{\widehat{S_i'}} = \begin{pmatrix} \widehat{\sigma}_{i_{T_{1+1}}} & \widehat{\sigma}_{i_{T_{1+2}}} & .... & \widehat{\sigma}_{i_{T_{2-1}}} & \widehat{\sigma}_{i_{T_2}} \end{pmatrix}$

it clearly emerges that the distribution of SAR becomes t-student with $\alpha - 2$ degree of freedom:

$\overrightarrow{\mathbf{SAR}}_i = \frac{\overrightarrow{\mathbf{AR}_i}}{\overrightarrow{\widehat{S'}_i}} \sim \underset{df=\alpha-2}{t_{student}}$

This statistic is built on the basis of the residuals of the regression line expressed by the market model. Since by construction $E(\overrightarrow{\mathbf{SAR}_i}(\Theta)) = 0$, if in the period Θ this hypothesis is not verified, the model defined in α will not explain the return of the ith stock in the period Θ and, therefore, the potential abnormal returns will be effectively abnormal ones.

7. aggregation on N stocks of the ith SAR. It is simply done by exploiting the SAR's distribution properties[12]. Indeed, it is sufficient to work on the r.v. average of the N $\mathbf{SAR}_i$ i.e.:

$\overline{\mathbf{SAR}} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{SAR}_i$

Moreover, since the sum of random variables normally distributed is still distributed normally and the mean of the sum equals the sum of the means, the r.v. $\overline{\mathbf{SAR}}$ is normally distributed too and $E(\overline{\mathbf{SAR}}) = 0$.

Hence, it follows that:

$\sqrt{N} \cdot \overline{\mathbf{SAR}} \sim \underset{df=\alpha-2}{t_{student}}$

8. hypotheses testing on the $\overline{\mathbf{SAR}}$ statistic in order to verify if the event occurrence has determined an abnormal return in the period Θ. Since, as explained above, this statistic entails the property of the model defined in the period α, the violation of its distribution property, i.e. $E(\overline{\mathbf{SAR}}) = 0$, will coincide with the rejection of the model in the period Θ and therefore with the conclusion that the events that occurred in the period Θ have determined an abnormality level in the return of the analyzed stocks.

The test is then constructed as follows:

$H_0 : E(\overline{\mathbf{SAR}}) = 0 \Rightarrow$ the events do not determine abnormal returns;

$H_A : E(\overline{\mathbf{SAR}}) \neq 0 \Rightarrow$ the events determine abnormal returns.

The null hypothesis H_0 will be rejected if $\left|E(\overline{\mathbf{SAR}}(\Theta))\right| > t_{\frac{\varkappa}{2}}$ for some prescribed $\varkappa$. $\varkappa$ is defined as the significance level in hypothesis testing problems. It represents the Type I error accepted in the test, that is the probability of rejecting H_0 when this hypothesis is true.

It is easy to compute the p-value, which is:

$p = P(E(\overline{\mathbf{SAR}}) \neq 0 \mid H_0 \text{ is true})$

If $p < \varkappa$ the null hypothesis is true at that significance level $\varkappa$, viceversa if the null hypothesis is rejected, this will confirm the presence of an abnormal return in the period Θ.

[12] The procedure shown hypothesizes that the stocks are non-correlated and that the event windows are not superimposed. Some straightforward computational adjustments are required to remove these hypotheses.

9. calculation of the cumulative abnormal returns in order to wholly represent the abnormality of the return over the period of analysis. To this end, it will be defined as the r.v. CAR (i.e. Cumulative Abnormal Return) given by the sum of the potential abnormal returns observed in the period Θ :

$\mathbf{CAR}_i = \sum_{j \in \Theta} AR_{ij}$

The CAR distribution is by construction:

$\mathbf{CAR}_i(\Theta) \sim N(0, \Theta \cdot V_i)$[13]

The aggregation on N stocks is particularly simple by exploiting the CAR's distribution properties[14]. It is sufficient to work on the average $\overline{CAR}$ statistics of the N $\mathbf{CAR}_i(\Theta)$;

$\overline{\mathbf{CAR}}(\Theta) = \frac{1}{N} \sum_{i=1}^{N} \mathbf{CAR}_i(\Theta)$

The $\overline{CAR}$ distribution is for construction:

$\overline{\mathbf{CAR}}(\Theta) \sim N(0, \overline{V})$

where:

$\overline{V} = \frac{1}{N^2} \sum_{i=1}^{N} \Theta \cdot V_i$

The graphical observation of this r.v., with respect to time, offers a clear and straightforward test of the abnormality of the returns over the period Θ. (figure 4.)

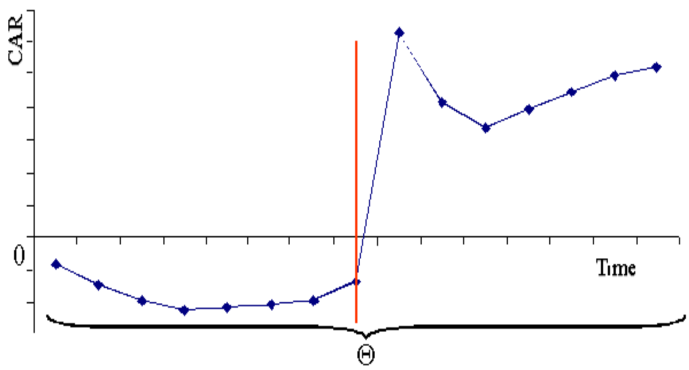

Figure 4:

In fact, it is easy to observe that the line that represents the $\overline{CAR}$, after the event occurred, a moment indicated with the vertical line, moves far from the zero value, increasing over time according to the value of the event and to its impact on the stock return.

[13] Also in this case the lack of a precise determination of V_i, as already emphasized, entails the employment of the estimator $\hat{V}_i$ consistent by construction.

[14] Also for this r.v. it is fundamental to hypothesize that the stocks are non-correlated and that the event windows are not superimposed.

Once it has been clarified how and why the event study is able to capture the economic value of the information of a company occurrence, in order to compute the disgorgement, it is important to understand how this theory has been used operatively in the US and in Italy.

3.1.2 The SEC methodology

The methodology described in the previous paragraph is applied with some simplifications by the SEC in order to analyze the insider-trading phenomena and specifically to calculate the disgorgement.

In particular, the methodology employed by SEC corresponds to the methodology explained before, without the aggregation of the different stocks, since the insider-trading investigation is carried out on the single case[15]. A short description follows which illustrates the modalities for the disgorgement calculation.

1. Individuation of the insider event Γ and of a $\Theta = 20$ days and a $\alpha = 120$ days[16].

2. Analysis of the company price evolution in the observation period in order to standardize data.

3. Calculation of the stock returns in α, as explained in point 2 of the previous paragraph.

4. Market model in the period α on the stock object of study:
$\underset{\alpha x1}{\mathbf{R}_i} = \underset{\alpha x2}{\mathbf{R}_m} \underset{2x1}{\beta} + \underset{\alpha x1}{\epsilon_i}$

5. Estimate of the vector $\underset{2x1}{\beta}$ with the least squared method.

6. Calculation of the potential abnormal return (AR) on Θ as previously explained, that is:
$\overrightarrow{\mathbf{e}_i} = \overrightarrow{\mathbf{AR}_i} = \overrightarrow{\mathbf{R}_i} - \mathbf{R}_m \overrightarrow{\hat{\beta}}$

7. Construction of the SAR statistic as explained in point 6 of the previous paragraph.

8. Hypothesis testing on the **SAR** statistic over the period Θ, in order to verify if the disclosure of the preferential information has determined some abnormality in the returns of the stock under investigation.

Since, as explained before, this statistic entails the property of the model defined in the period α, the violation of its distribution property, i.e. $E(\mathbf{SAR}) = 0$, will coincide with the rejection of the model in period Θ and therefore with the conclusion that the disclosure of the inside information that occurred in period Θ has determined an abnormality level in the return of the investigated stock.

The test is then defined as follows:

[15] This simplification is not trivial from a statistical point of view since it could create some convergency issues in the probability distribution of the stock return and hence in that of the SAR r.v.. Particularly, as is more clearly explained in the paragraph describing the CONSOB methodology, this choice combined with some market issues could violate *a priori* the statistical properties entailed by the model and hence it could render the disgorgement calculation meaningless.

[16] As seen in the previous paragraph, Θ is a time period that crosses the insider event, while α is defined before the preferential information is disclosed to the market.

$H_0 : E(\mathbf{SAR}) = 0 \Rightarrow$ the inside information does not determine abnormal returns;

$H_A : E(\mathbf{SAR}) \neq 0 \Rightarrow$ the inside information determines abnormal returns;

As in the previous paragraph, the significance level $\varkappa$ to test the hypotheses will be defined, and the p-value and the cumulative abnormal return in order to graphically represent the abnormality level of the stock return analyzed can be computed.

9. The computation of the disgorgement consists in simply multiplying the abnormal return by the quantity involved in the insider transactions. Obviously, this computation will proceed if and only if the hypothesis testing confirms that the preferential information has determined an abnormality in the return.

3.1.3 The CONSOB methodology

The SEC methodology cannot be applied as it is to the Italian market. This is because of the peculiarities of the Italian market, such as:

i. the lack of liquidity of most listed stocks;

ii. the large presence of companies recently listed on the stock exchange;

iii. the empirical observation of some seasonality effects.

The methodology can be exemplified in the following fundamental passages:

1. Individuation of the insider event Γ defined as Γ_2.

2. The definition of α and Θ. As regards Θ it is fixed equal to 20 days, as in the US procedure. Because of the Italian market features, the choice of α has to be made with specific accuracy. For instance, the absence of liquidity of the quoted stocks implies that 120 observations would not be enough to ensure the statistical significance of the model and the presence of seasonality in market trends implies that the enlargement of the time window could include non-homogeneous observations.

There follows an explanation of the solution, adopted by CONSOB, which was developed in order to overcome these issues.

The time horizon α is defined around 600 days[17]. This choice is supported by some empirical analyses of the Italian Stock Market Index (i.e. MIB) returns in connection with their convergence in distribution towards the standard normal r.v. by the central limit theorem. In these analyses the returns of n days with n variables have been aggregated in order to determine the value which leads to a random variable normally distributed with parameters μ and σ. This can be represented in formulae as follows[18]:

$$Find\ n \because\rightarrow P\left\{\frac{\ln\frac{S_1}{S_0}+\ln\frac{S_3}{S_2}+.....+\ln\frac{S_{n+1}}{S_n}-n\mu}{\sigma\sqrt{n}} \leq a\right\} \longrightarrow \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{a} e^{-\frac{x^2}{2}}dx$$

So far, the procedure has overcome the first two problems related to the Italian market. Regarding the seasonality phenomena observed in stocks listed in the Italian financial market, the methodology proceeds on broken single time

[17] The choice of such a wide α makes it more suitable to analyse if there has been some extraordinary events for the company which could have generated some issues in the data set considered.

[18] The results of these empirical analyses are available from the author on request.

windows rather than directly on one wider single time window. In particular, it identifies the days Γ_1, Γ_0, as the same dates of the event Γ_2 in the two previous years. Hence, α becomes a vector: $\overrightarrow{\alpha} = (\alpha_0, \alpha_1, \alpha_2)$ where $\alpha_0, \alpha_1, \alpha_2$, equal to 200 days each, are the time windows before $\Gamma_2, \Gamma_1, \Gamma_0$. This vector eventually defines 3 periods for a total of 600 observations.[19]

3. Market model on the stock object of study by using the MIB index, the Italian and the European sector indexes as regressors.

a) $\underset{\alpha x1}{\mathbf{R}_i} = \underset{\alpha x2}{\mathbf{R}_{MIB}} \underset{2x1}{\beta} + \underset{\alpha x1}{\epsilon_i}$

b) $\underset{\alpha x1}{\mathbf{R}_i} = \underset{\alpha x2}{\mathbf{R}_{\text{Italian sector index}}} \underset{2x1}{\beta} + \underset{\alpha x1}{\epsilon_i}$

c) $\underset{\alpha x1}{\mathbf{R}_i} = \underset{\alpha x2}{\mathbf{R}_{\text{European sector index}}} \underset{2x1}{\beta} + \underset{\alpha x1}{\epsilon_i}$

4. Comparison of the results that emerged from the linear regression model mentioned in the previous point and verification that the regression analysis respects the key assumptions of the model.[20]

On the basis of the results of these analyses the index with the highest statistical robustness will be chosen.[21]

The exemplification of the diagnostic measures adopted by CONSOB in making this choice follows[22]. It was decided to proceed with graphic-type diagnostic measures; in particular, it is possible to act, with reference to the independency of regressor observations, through the sequence plot; this represents the regressor values in connection with time; the non-recognizability of a pattern suggests the above-mentioned independence (figure 5).

As far as the distribution properties of the errors, it is necessary to verify that these are normally distributed with constant variance; the normality of errors can be easily diagnosed by means of the error histogram analysis, or of the qqplot; the first one represents the statistical distribution of errors and therefore allows the verification of the approximation of this diagram with the typical bell-shaped one of the normal distribution(figure 6); the second one draws a 45°line which represents the quantiles of normal distribution; the higher the concentration of the observations identifying errors around this line the higher the normality of distribution (figure 7).

With reference to the variance constancy, it is necessary to proceed with the analysis of the *residual vs fitted* diagram; this diagram represents errors (residuals) related to the *fitted*; the presence of a clear band of observations guarantees the constancy of the variance V_i. Viceversa, the determination of a possible sinusoidal oscillation as shown in figure 8 is a premonitory sign of heteroskedasticity of the variance.

[19] The proposed solution has shown robust empirical evidence. The results of these empirical analysis are available from the author on request.

[20] See point 3 of paragraph 3.1.1..

[21] For a thorough explanation of the criteria used for the choice of the regressor, see Greene, W.H., (1993) Econometric Analysis, Prentice Hall.

[22] For an exaustive discussion of these statistical measures, see Neter, J., Kutner, M., Nachtsheim, C., Wasserman, W., (1996) Applied Linear Regression Models, The McGraw-Hill Companies, Inc.

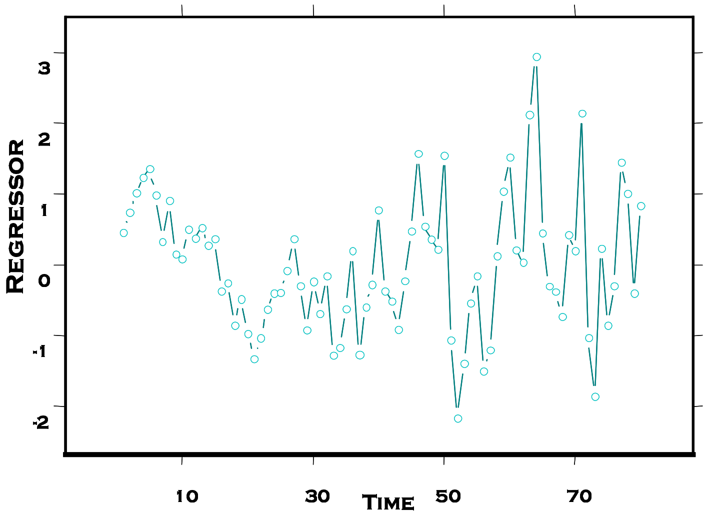

Figure 5:

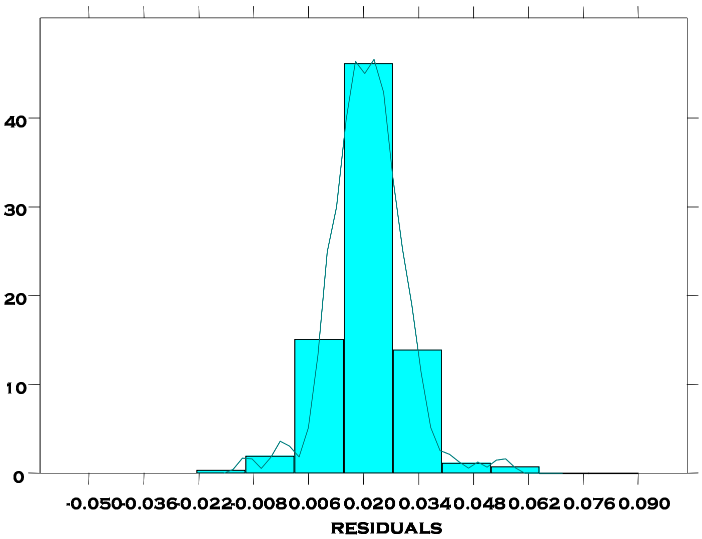

Figure 6:

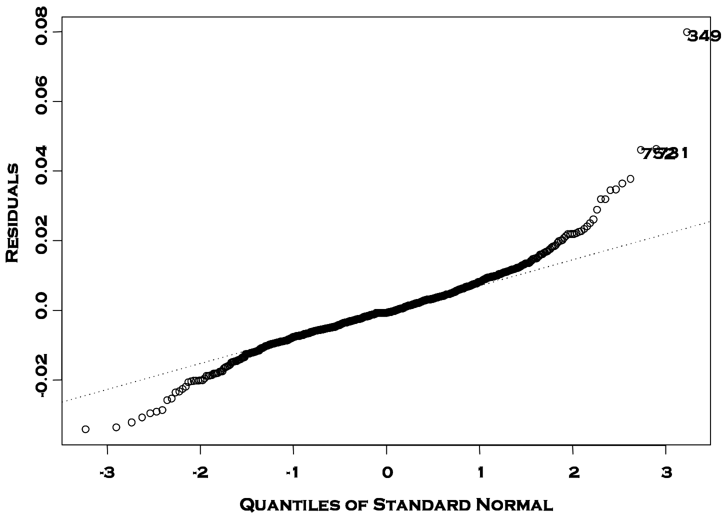

Figure 7:

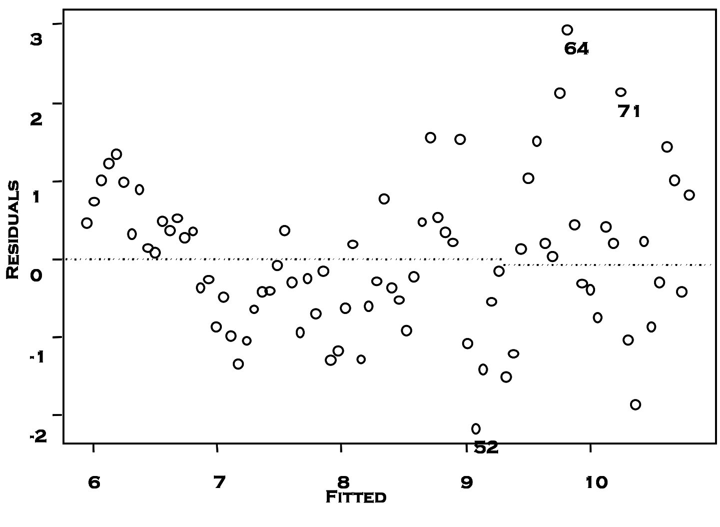

Figure 8:

The graphical analyses can be put together with numerical diagnostic tests, such as the Levene or Breusch-Pagan test.

As far as the errors independency is concerned, the residual vs time diagram which represents the residuals in connection with time can be analysed; the non-recognizability of a pattern guarantees the above-mentioned independence. (figure 9)

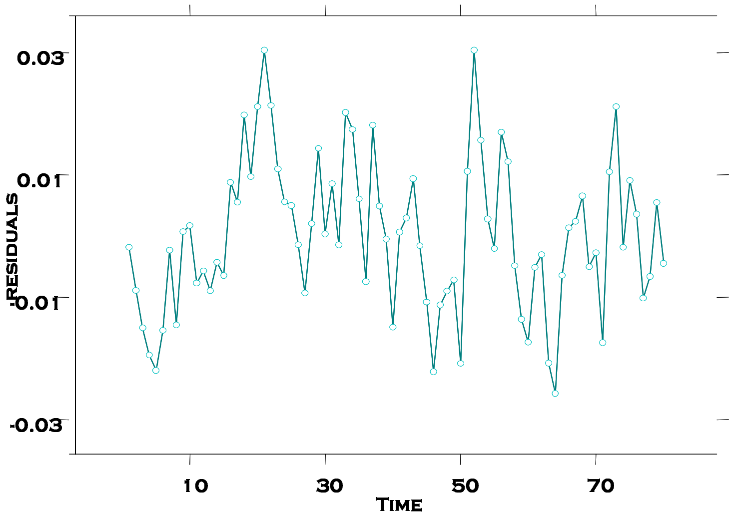

Figure 9:

5. Estimation of vector $\underset{2x1}{\beta}$ with the least squared method: $\underset{2x1}{\widehat{\beta}^{\overrightarrow{\alpha}}}$. Moreover, in order to verify a recursive stability of parameters, there is the computation of parameters $\underset{3x2}{\widehat{\beta}'} = \begin{pmatrix} \widehat{\beta}_0^{\alpha_0} & \widehat{\beta}_1^{\alpha_0} \\ \widehat{\beta}_0^{\alpha_1} & \widehat{\beta}_1^{\alpha_1} \\ \widehat{\beta}_0^{\alpha_2} & \widehat{\beta}_1^{\alpha_2} \end{pmatrix}$ of the market model respectively for the three elements of the vector $\overrightarrow{\alpha}$. The constancy of parameters in the three periods, object of the three regression analyses, guarantees the reliability of the results of the statistical analysis. This verification can also be carried out through statistical tests, such as the Chow test, or graphical analyses, such as the Recursive beta diagram[23].

6. Calculation of the potential abnormal return on Θ, as previously seen that is:

$$\overrightarrow{\mathbf{e}_i} = \overrightarrow{\mathbf{AR}_i} = \overrightarrow{\mathbf{R}_i} - \mathbf{R}_m \overrightarrow{\widehat{\beta}^{\overrightarrow{\alpha}}}$$

The calculation of the cumulative abnormal return, the statistics construction, the hypothesis testing, and the disgorgement determination are carried out as envisaged in the SEC procedure.

[23] For an exaustive discussion of these tests see Greene, W.H., (1993) Econometric Analysis, Prentice Hall.

3.2 Problems

Both procedures have structural weaknesses which can be explained as follows:

1. the methodology requires a time series data set that may not be available if the stock has been recently quoted on the stock exchange;

2. the insider-trading investigation is subordinated to the determination of a reference index that is statistically meaningful as a regressor and to the determination of a market portfolio model (proxy). This investigation is not easy for any financial market; in particular for the Italian market the presence of a high number of thin stocks hampers the implementation of the model. Moreover, the fact that there are some stocks which account for the bulk of the market reference index can determine that the results of the regression analysis would appear to be statistically meaningful while they actually reflect a self-explanatory regression since the relationship between the two variables is endogenous;

3. the employment of a particularly long time horizon could include phenomena which have changed the company capitalization, and it must be specified that the data homogenization methodologies are biased and difficult to support statistically; this happens because of the lack of a standard reference behavior of the stock market in the event of regulation variation, or of mergers and acquisitions occurrences;

4. the event study applied to insider-trading investigation determines the future trend of stock returns with a linear regression model. It is therefore based upon the assumption that these returns on a narrow interval Θ are generated by the same linear model on the basis of parametric coefficients coming from a set of information belonging to a definitely wider time window α. Hence, what has been said above breaks down the thesis of the weak form of market efficiency, which states the impossibility of predicting the future on the basis of deterministic models, which are founded on sets of information belonging to the past, since the stock prices in the present already contain the information of the past;

5. the results of the parameter time stability analysis are discriminating for the statistical investigation of the insider-trading case; in other words, if the parameter stability is not verified, the research into the phenomenon can not continue without inevitable methodological problems;

6. often rumors on the stock generate spikes on the return in the period α, time reference for the parameter estimate;

7. the methodology does not consider in the computation the insider-trading strategy, which usually should reflect the insider knowledge level about the value of the preferential information. In fact, it applies to all insider-trading cases involved in the investigation the same abnormal returns computed in relation to a specific preferential information regardless to the closeness of the insider to the event;

8. with reference to SEC methodology, the time horizon of 120 days is not necessarily sufficient for a time series analysis and in particular to frustrate

the second term of V_i, which determines, as already said, serial correlation phenomena; therefore, the regression results become invalid and statistically unreliable. Even if the usage of statistical methodology (e.g. first difference of the return) may solve the issue of autocorrelation in the period α, it is not certain that the same technique is valid in the time horizon Θ.

In order to tackle all these problems, a new methodology, adopted by CONSOB, has been developed on the basis of the probabilistic theory, which allows the discovery of the economic value of the information exploited by each insider. This procedure has been defined, as stated before, as *potential probabilistic disgorgement.*

3.3 Potential Probabilistic Disgorgement

3.3.1 The new approach adopted by CONSOB

What is proposed as an alternative to the model derived from the event-study analysis is a probabilistic model which simulates the stock trend in time through a stochastic differential equation. Stochastic modelling has also been applied to insider-trading analysis in order to identify abnormal trading in the financial markets. (Grorud and Pontier, 1998). The stochastic differential equation chosen is known in probability as geometric Brownian motion and has been used in finance by Black-Scholes (1973) for their well-known option pricing model:[24]

$dS_t = \mu S_t dt + \sigma S_t dW_t$ [1]

The solution of this equation is:

$S_t = S_s \cdot e^{\left(\mu - \frac{\sigma^2}{2}\right)(t-s)+\sigma(W_t - W_s)}$ *where* $s \leqslant t$ [2]

which describes in the continuum the price fluctuation of the single stock S (figure 10).

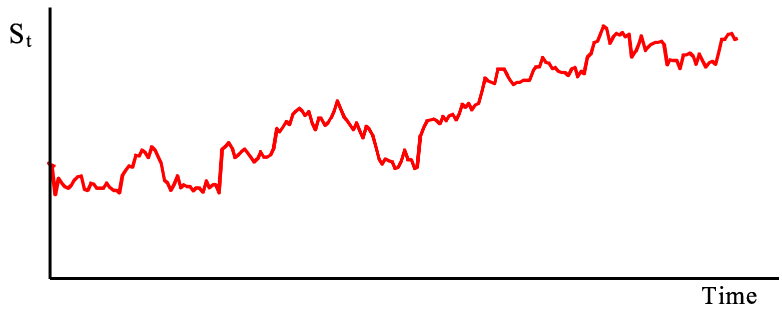

Figure 10:

[24] An in depth analysis of the main features of the equation [1] and the demonstration that [2] is the only admissable solution of [1] is developed in appendix B.

With this solution it is possible to simulate the path that the stock price will follow in the future by employing the current position of the stock itself and hypothesizing a logarithmic stock-return increase rate equal to $\mu - \frac{\sigma^2}{2}$ and a dispersion in this rate quantified in the parameter σ. (figure 11)

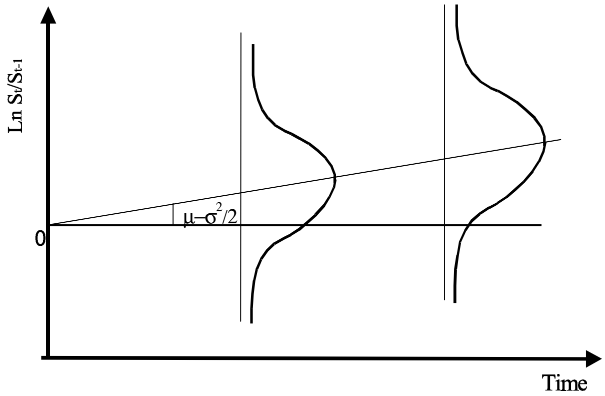

Figure 11:

This equation benefits from the strong Markov property:
$P(S_{T+1} = X \mid S_T, S_{T-1}, S_{T-2}, S_{T-3}, ..., S_0) = P(S_{T+1} = X \mid S_T)$.
In other words, the probability that the stock price variable takes a certain value X in the future, considered the values it has assumed until the present, is equal to that conditioned only on the present. This property is then absolutely coherent with the weak form of market efficiency.

This model complies with the normal probability distribution of the logarithmic stock returns:[25]
$\ln \frac{S_t}{S_s} \backsim N\left(\left(\mu - \frac{\sigma^2}{2}\right)(t-s), \sigma\sqrt{(t-s)}\right)$

The new methodology proposed borrows the definition of the two time horizons α and Θ from the event-study analysis, but it defines them in a different way. α corresponds to the period in which the insider will build his position on the stock[26]. Θ is no longer a period which contains the event but it is defined by the day in which the event information is given and the first or the second or the n^{th} day after, according to the liquidity of the stock under investigation[27] (figure 12).

The hypotheses behind these choices are that the insider:

i. cannot control what happens to the price stock dynamic before the

[25] This distribution property is demonstrated in appendix B.

[26] In a standard insider-trading scheme, it usually lasts for a period that goes from 5 to 15 days before the release of the information.

[27] In a standard insider-trading scheme, it coincides with the moment in which the insider closes his position.

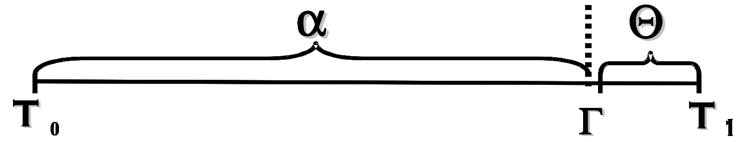

Figure 12:

event (i.e. the insider is a price taker)[28]. This is mainly because the insider does not want to risk having his trading recognized as insider by the market. In other words he wants to hide his insider-trading strategy.

ii. draws his operative strategy on the stock in the period α. The insider, in a context of a hit-and-run strategy, will create a long (short) position on the stock, if the event information will have a bullish (bearish) effect on the price stock trend.

Therefore, the insider creates his position in the period before the information is given (i.e. the period α), and since he knows the value of the information, it clearly emerges that he will only gain if the information generates a higher price than the one he has incorporated in his portfolio and in a certain sense more volatility than in the period α, in which he has built his position. In other words, the insider will make a profit if the information is so price sensitive as to absorb the price oscillation that the stock has shown in the period α. What is stated above means that the insider forecast about the stock price dynamic in the period Θ is that the information will move it more than it moved in the period α. But in terms of the stochastic methodology proposed, it means that the insider makes his profit forecast based on the μ and σ determined in the period α, in which he has created his position. Consequently the right parameters to replicate the correct price stock dynamic in the model and to quantify the insider-trading disgorgement are the parameters that the insider hypothesizes and hence incorporates in his portfolio strategy. (i.e. the μ and σ in the period α).

More formally, the model defines a probability measure Q in a continuous trading economy with a finite horizon $t \in \alpha$. The uncertainty in this economy is classically modelized by a complete probability space (Ω, F, Q) and it depends on the value of the information the insider would appropriate. This value evolves according to the augmented filtration $\{F_t, t \in \alpha\}$ generated by a one dimensional geometric brownian motion $(S_t)_{t \in \alpha}$.

Moreover, as shown in figure 13, every insider (i.e. Insider A or Insider B), according to his closeness to the preferential information, will have a different strategy, and hence a different period α, since it will give a different value to the information. This will imply that every insider will have its own probability

[28]If the insider would be able to influence on the stock price dynamics, it will be a trade based manipulation case and not an insider trading investigation.

measure Q and the value of the information will evolve according to a different augmented filtration F. This choice for the parameters estimation should allow the model to represent the value of the information for different insiders in a more realistic way.

Therefore, by construction, this model attributes a higher disgorgement to the insider who has the better strategy. More precisely, the filtration $\{F_t, t \in \alpha\}$ governing the stock price dynamics will continuously represent the level and quality of information the trader has previously acquired. The more precise the information the trader possesses the more likely he will choose an α time period which will allow him to implement an optimal trading strategy. Hence, the filtration $\{F_t, t \in \alpha\}$ will be reflected in the parameters governing the stock price stochastic differential equation in the latter period Θ, ensuring minimal volatility (i.e. maximal potential probabilistic disgorgement) to the trader who has complete and immediate access to preferential information. In fact the best strategy for the model is the one which defines the probability space (Ω, F, Q) with lowest drift $\mu - \frac{\sigma^2}{2}$ and lowest dispersion rate σ. This solution, by hypothesizing that whoever is closer to the information should have the more profitable strategy, is able to distinguish between insiders and followers like tippes and other non-institutional insiders.

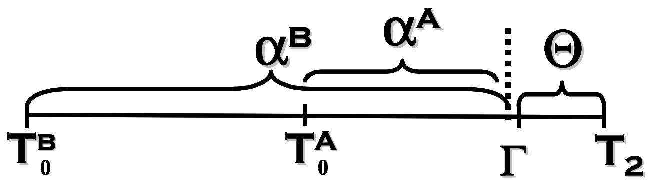

Figure 13:

On the basis of the augmented filtration $\{F_t, t \in \alpha\}$, it is possible to determine an oscillation band for the price of the stock under investigation. If there were no such event occurrence, the stock price would evolve remaining in this band. This is because the insider investment strategy has been defined according to the value of the information, to its price sensitivity and, what is more to the fact that the information is not available to other investors. Therefore the price dynamic incorporated in the insider portfolio defines the future price evolution of the stock if the information had never existed (figure 14).

The difference between the actual stock price after the insider information is disclosured to the market (i.e. the period Θ) and the band will therefore represent the value of the information that the insider trader would appropriate, i.e. the digorgement (figure 15-16).

Moreover, as explained before, each insider has a different investment strategy, according to his knowledge of the fraudolent information, and therefore a different stock price oscillation band and eventually a different disgorgement.

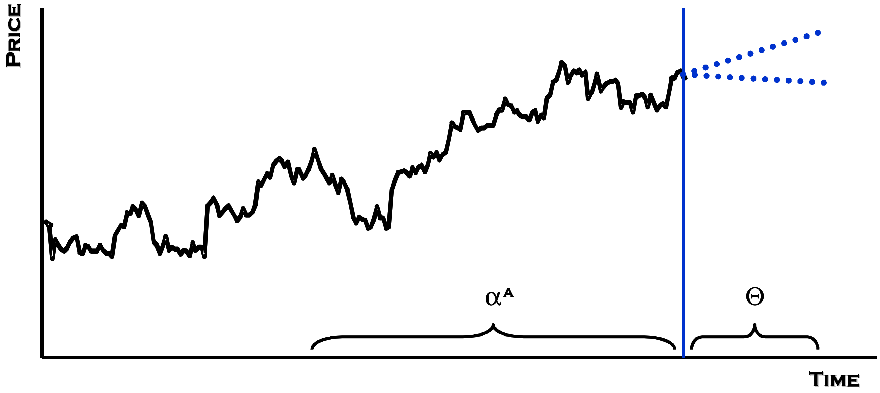

Figure 14:

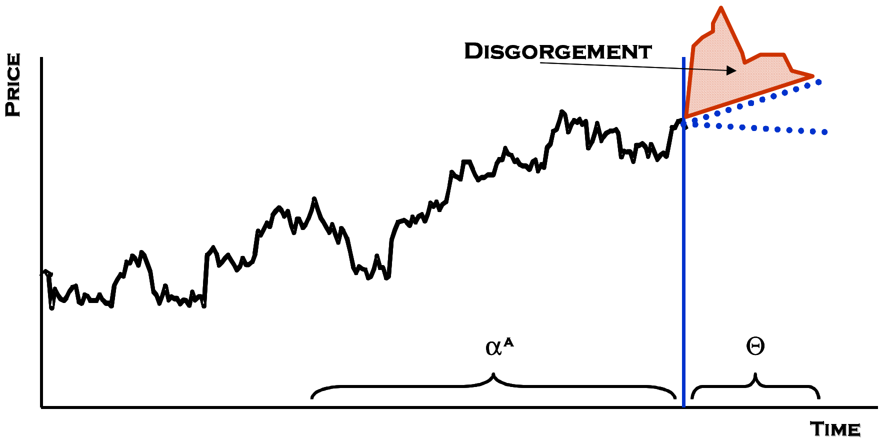

Figure 15:

The disgorgement for Insider A is shown in figure 15 while the value for Insider B is shown in figure 16.

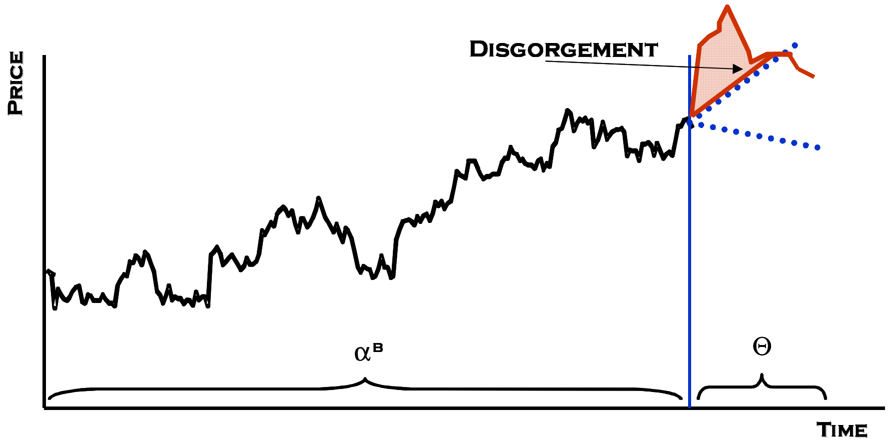

Figure 16:

In operative terms the model develops in the following stages:

1. Determination of the periods α and Θ.

2. Verification that there are no structural events in these periods that move the whole market hugely.[29]

3. Individuation of the probability space (Ω, F, Q) which represents the strategy of the insider in the period α.[30].

4. Individuation of an oscillation band for the prices of the stock object of study in every t^{th} day of the period Θ by assuming that the stock will evolve according to the augmented filtration $\{F_t,\, t \in \alpha\}$ generated by the one dimensional geometric brownian motion $(S_t)_{t\in\alpha}$ identified in the previous point:

$\Delta\widehat{S}_t^{\Theta} = [S_o^{\Theta} e^{\max}, S_o^{\Theta} e^{\min}]$ [3]

where

$max = \sigma z_{\frac{\varkappa}{2}} \sqrt{t} + \left(\mu - \frac{\sigma^2}{2}\right) t$

$min = \sigma \left(-z_{\frac{\varkappa}{2}}\right) \sqrt{t} + \left(\mu - \frac{\sigma^2}{2}\right) t$

S_o is the price of the stock[31];

$z_{\varkappa}$ is the value of the probability density function of a standard normal random variable; by fixing $\varkappa$, we identify the probability that the

[29] In these cases, some preliminary analyses have to be developed in order to quantify the effects of these exogenous events and to try to purge the stock price trend under investigation. Although it has to be considered that if the stock price trend is dramatically changed by structural events, it is reasonable to hypothesize that the insider strategy is going to break down.

[30] The time reference for the computation will be daily.

[31] In a standard insider-trading scheme, this coincides with the last day before the event information.

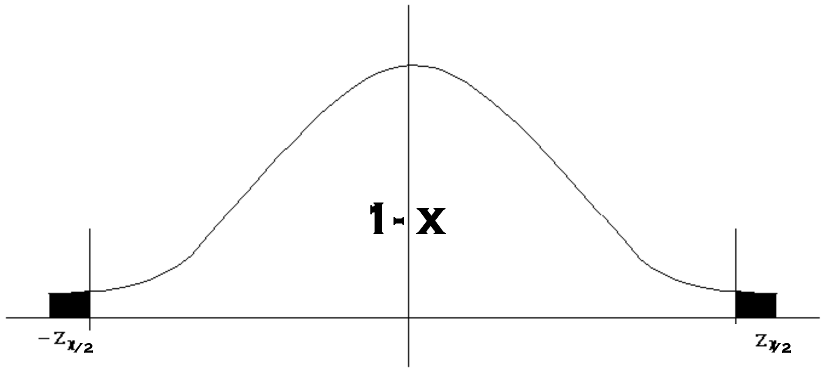

Figure 17:

standard normal r.v. Z will lie between the interval $[-z_{\frac{\varkappa}{2}}, z_{\frac{\varkappa}{2}}]$, i.e. $P(-z_{\frac{\varkappa}{2}} \leq Z \leq z_{\frac{\varkappa}{2}}) = \varkappa$. In other words, the definition of $\varkappa$ determines the percentage of price evolution scenarios included in the price band. For instance, $\varkappa = 2.5\%$ means that the band will include the 97.5% of all the possible price scenarios (figure 17).

The band determination emerges from the following hypotheses [32]:

i. $S_t = S_s \cdot e^{\left(\mu - \frac{\sigma^2}{2}\right)(t-s)+\sigma(W_t - W_s)}$

ii. $\ln \frac{S_t}{S_s} \backsim N\left(\left(\mu - \frac{\sigma^2}{2}\right)(t-s), \sigma\sqrt{(t-s)}\right)$

iii. $P\left(S_s e^{\min} \leq S_t \leq S_s e^{\max}\right) = \varkappa$[33]

By setting s equal to 0, in the three considerations given above, the band for the generic day $t \in \Theta$ becomes the interval expressed in [3].

5. Verifying whether prices in the period Θ lie within the oscillation band or not.

6. Determination of the Abnormal Return as:

$$AR_t^{\Theta} = \left(S_o^{\Theta}\right)^{-1} \cdot \max\left[0, sign\left(S_t^{\Theta} - S_o^{\Theta} e^{\max}\right) \cdot sign\left(S_t^{\Theta} - S_o^{\Theta} e^{\min}\right)\right] \cdot \\ \cdot \min\left[\left|\left(S_t^{\Theta} - S_o^{\Theta} e^{\max}\right)\right|, \left|\left(S_t^{\Theta} - S_o^{\Theta} e^{\min}\right)\right|\right]$$

where the *sign* function gives back 1 (-1) if its content is positive (negative).

7. Determination of disgorgement as the quantity involved every day of the period Θ in the insider trading multiplied by its correspondent abnormal return.

3.3.2 The advantages of the potential probabilistic disgorgement computation

On the basis of the methodological considerations explained and of the analysis of the model fundamental characteristics, there follows the recapitulation of the advantages offered by the probabilistic approach adopted by CONSOB:

[32] A comprehensive examination of the hypotheses is shown in appendix B.

[33] As mentioned before, the choice of $\varkappa$ determines how many price evolution scenarios will be included at the generic time t in the price oscillation band $\Delta\widehat{S}_t^{\Theta}$.

1. the definition of the parameters is extremely realistic and difficult to break down, since it represents the insider-trading strategy on the stock under investigation carried out in the period α;

2. it allows the determination of all the possible paths of the stock under investigation under a predictive dynamic logic;

3. it cannot be invalidated by the fact that the company has been recently quoted, since if the insider can trade the stock, the procedure can return, by means of the parameters-estimation procedure, a disgorgement computation for him;

4. it does not require a regressor since the stock path forecast depends only on the prices of the stock under investigation incorporated in the insider trader portfolio;

5. it does not require the definition of time horizons longer than 15 days for estimating the parameters to be employed in the analysis;[34] therefore, it is not affected by the stock liquidity, by the discontinuity of the time series, and other typical issues of econometrical procedures;

6. thanks to the parameters-estimation procedure, it offers a sort of customized methodology for the single subject under investigation, since the model, by construction, behaves differently according to the single insider-trading strategy; moreover, by assuming that the insider who is closer to the information will have the more profitable trading strategy, it gives a higher disgorgement to the subjects who are closer to the preferential information and therefore it is able to distinguish between insiders and followers (i.e. tippes and other insiders);

7. the computation of the disgorgement is more conservative since, instead of using the cumulative abnormal return, it is determined by directly multiplying the Abnormal return of the t^{th} day of the period Θ by the correspondent quantity of stocks involved in the insider trading; by doing so the model also considers the ability of the stock to absorb the information;

8. it complies with the normal distribution property of the logarithmic stock returns;

9. the stochastic process employed benefits from the Markov property. This property makes the model absolutely coherent with the weak form of market efficiency;

10. finally, from an operative point of view:

a. it is a more intuitive approach, since it works directly on prices and not on return; moreover the reversibility between these two quantitative measures is straightforward to compute;

b. it is a faster and easier procedure, in terms of implementation, than the potential econometric disgorgement computation, since it can skip all the issues related to the statistical robustness tests.

[34] As we have expained before, usually the insider trading strategy does not last more than 5-15 days.

4 Conclusions

The quantitative methodologies related to the analysis of insider trading are used in order to detect the phenomenon and calculate the disgorgement, which is the undue enrichment gained by the insider through the exploitation of the preferential information.

The detection phase of the insider affects the level of sensitivity in the market analysis carried out by the supervisor, that is the amount of signals to put under scrutiny.

The evaluation of the disgorgement affects, in all the legal systems punishing the crime of insider trading, the sanction imposed against the insider and in this sense it can be considered as the linking point between the financial and legal aspects.

Therefore, the supervisors expend a great deal of effort in the attempt to define an accurate estimate of the value of the information exploited by the insider.

This paper presents the different methodologies developed in this field. In particular, it shows that the traditional method which computes the disgorgement as the profit gained by the insider does not work, as the insider strategy is hard to reduce to a simple scheme.

Therefore, the econometric procedure developed by the SEC represents an innovative and successive attempt to produce an objective measure of the value of the information. In particular, the paper shows how this methodology could be fitted to the different features of each financial market, by developing an adaptation for the Italian one. It demonstrates that the potential-econometric-disgorgement computation has upgraded the procedure relating to the evaluation of the profit gained by the insiders, but it still has some structural weaknesses such as the need of a long time series data set and of a statistically robust regressor. Moreover, it has been demonstrated that these issues can completely invalidate the working of the procedure.

This work presents a new approach for the analysis of insider-trading cases and the computation of the disgorgement (different from the traditional event studies methods). It uses probabilistic procedures and allows the analysis of the shifts in price of the securities in the financial markets based on the current stock price and on an analysis of all the future scenarios, giving them a suitable probability measure. Furthermore, the use of the stock prices incorporated in the insider-trading strategy to determine the value of the parameters of the model implies that the procedure can apply to all insider-trading schemes and it is unlikely to break down from a statistical point of view.

The potential-probabilistic-disgorgement computation provides a solutions for the problems affecting the traditional event-studies methodology, such as the individuation of the market proxy portfolio, the need for a long time series data set, the temporal stability of the regression parameters and the consistency of the linearity and deterministic relation among the variables of the model. Moreover, it is able to identify a specific disgorgement for each insider according to his trading strategy, instead of applying, as in the econometric approach, one unique value (i.e. the abnormal return) for all the insiders related to the same preferential information.

In addition, the use of probabilistic models in finance has been corroborated in the most recent empirical analyses and the workings of the Intermediaries are increasingly based on the use of quantitative methodology as a competitive hedge to make profit and reduce and unbundle financial risk.

In a world where effectiveness of supervision means taking enforcement action in order to protect the investors and to guarantee the efficiency and the integrity of the financial system, while at the same time avoiding acting as a constraint for the system itself, the use of quantitative methodology in the enforcement process could be the solution to achieve both these targets.

Within this framework, this paper, which develops a probabilistic approach to dealing with a supervisory issue from the point of view of the regulators, can be considered a contribution to the current debate on the need to regulate, enforce and supervise by using quantitative methodologies.

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Appendix A

Glossary:

SEC: United Stated Securities and Exchange Commission.

CONSOB: Italian Securities and Exchange Commission.

Bullish(bearish) information: information that, when announced to the market, will move up (down) the stock price.

event, occurrence: fact that changes the value of the company.

insider (trader): investor who accomplish an insider trading strategy.

insider trading (strategy): stock trading that is based on the exploitation of a preferential information.

disgorgement: the undue enrichment of the insider connected to the exploitation of the preferential information.

preferential / *inside* / *insider* / *fraudulent* / *price sensitive* *information:* information about a quoted company that influences its market price.

AR: Potential Abnormal Return.

SAR: Standardized Potential Abnormal Return.

CAR: Cumulative Abnormal Return.

Appendix B

Value of the parameters of r.v. AR

$\overrightarrow{\mathbf{AR}_i} \overset{?}{\backsim} N(0, V_i)$

i.e.:

i. $E(\mathbf{AR}_i \mid \mathbf{R}_m) \overset{?}{=} 0$

ii. $V_i \overset{?}{=} I \cdot \sigma_i^2 + E(\mathbf{R}_m(\widehat{\beta} - \beta)(\widehat{\beta} - \beta)' \mathbf{R}'_m \mid \mathbf{R}_m)$

with reference to the mean:

$E(\mathbf{AR}_i \mid \mathbf{R}_m) = E(\mathbf{R}_i - \mathbf{R}_m\widehat{\beta} \mid \mathbf{R}_m)$

adding and subtracting $\mathbf{R}_m\beta$, the result is:

$= E((\mathbf{R}_i - \mathbf{R}_m\beta) - \mathbf{R}_m(\widehat{\beta} - \beta) \mid \mathbf{R}_m)$

$= E((\mathbf{R}_i - \mathbf{R}_m\beta) \mid \mathbf{R_m}) - \mathbf{E}(\mathbf{R}_m(\widehat{\beta} - \beta) \mid \mathbf{R}_m)$

given the consistency properties of the estimator $\widehat{\beta}$ the second term disappears.

The first term can be written as follows:

$= E(\mathbf{R}_i \mid \mathbf{R_m}) - E(\mathbf{R}_m\beta \mid \mathbf{R_m})$

given the regression model hypothesis:

$E(\mathbf{AR}_i \mid \mathbf{R}_m) = 0$

q.e.d.

with reference to the variance:

$V_i = E(\underset{\Theta x1}{\mathbf{AR}_i} \cdot \underset{\Theta x1}{\mathbf{AR}'_i} \mid \mathbf{R}_m) =$

rendering explicit:

$= E((\mathbf{R}_i - \mathbf{R}_m\widehat{\beta})(\mathbf{R}_i - \mathbf{R}_m\widehat{\beta})' \mid \mathbf{R}_m)$

adding and subtracting in every term ϵ_i:

$= E((\epsilon_i - \mathbf{R}_i + \mathbf{R}_m\beta + \mathbf{R}_i - \mathbf{R}_m\widehat{\beta})(\epsilon_i - \mathbf{R}_i + \mathbf{R}_m\beta + \mathbf{R}_i - \mathbf{R}_m\widehat{\beta})' \mid \mathbf{R}_m)$

opening brackets:

$= E((\epsilon_i\epsilon'_i - \epsilon_i(\widehat{\beta} - \beta)' \mathbf{R}'_m - \mathbf{R}_m(\widehat{\beta} - \beta)\epsilon_i + \mathbf{R}_m(\widehat{\beta} - \beta)(\widehat{\beta} - \beta)' \mathbf{R}'_m \mid \mathbf{R}_m)$

simplifying:

$= E((\epsilon_i\epsilon'_i \mid \mathbf{R}_m) + E(\mathbf{R}_m(\widehat{\beta} - \beta)(\widehat{\beta} - \beta)' \mathbf{R}'_m \mid \mathbf{R}_m)$

simplifying:

$V_i = I \cdot \sigma_i^2 + E(\mathbf{R}_m(\widehat{\beta} - \beta)(\widehat{\beta} - \beta)' \mathbf{R}'_m \mid \mathbf{R}_m)$

q.e.d.

Solution of equation [1]

The stochastic differential equation:

$dS_t = \mu S_t dt + \sigma S_t dW_t$ [1]

has the only admissible solution:

$S_t = S_s \cdot e^{\left(\mu - \frac{\sigma^2}{2}\right)(t-s) + \sigma(W_t - W_s)}$ *where* $s \leqslant t$ [2]

First of all this equation shall have a *strong* solution. For this reason it is sufficient to verify that the following inequality is true:

$|\mu x - \mu y| + |\sigma x - \sigma y| \leq (|\mu| + |\sigma|) \cdot |x - y|$

The term $(|\mu| + |\sigma|)$ can be considered as a positive generic constant D and therefore the inequality is evidently true. We can conclude that this solution is unique since the Lipschitz coefficients of the differential stochastic equation are continuous.

When applying the Ito rule the result is the above mentioned solution:

$dS = \frac{dS}{dt}dt + \frac{dS}{dW}dW_t + \frac{1}{2}\frac{d^2S}{dW^2}dt$ [4]

Then there is the computation of the derivatives:

$S = f(t, x) = S_s e^{\left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dW_t}$

$\frac{dS}{dt} = \left(\mu - \frac{\sigma^2}{2}\right) S$

$\frac{dS}{dW} = \sigma S$

$\frac{d^2S}{dW^2} = \sigma^2 S$

Substituting in [4] the derivatives we obtain:

$dS = \left(\mu - \frac{\sigma^2}{2}\right) Sdt + \sigma dW_t + \frac{1}{2}\sigma^2 Sdt$

simplifying:

$dS = \mu Sdt + \sigma SdW$

For what has been written, we can conclude that [2] is the only admissible solution of [1].

q.e.d.

Brownian Motion and Geometric Brownian Motion properties

W is called standard Brownian motion. This stochastic process has the following properties:

i. $W_0 = 0$

ii. $(W_t - W_s) \backsim N(0, (t - s))$i.

iii. $(W_{t_2} - W_{t_1})$ is independent from $(W_{t_3} - W_{t_2})$ where intervals $[t_3 - t_2)$ e $[t_2 - t_1)$ do not superimpose;

iv. benefits from the strong Markov Property, that is:
$P(W_{s+t} \in c \mid W_t = x, W_0 = y) = P(W_{s+t} \in c \mid W_t = x)$

v. is a continous function in time, that is with any w we have:
$\{W_t(w)\}_{t \geq 0}$ for $t \geq 0 \because W_t$ is continous that is:
$P((w) \because t \longrightarrow W_t(w) \text{ is continuous}) = 1$

It is easy to demonstrate that also $W_{t+s} - W_t = \Delta W_t \triangleq B_s$ is a Brownian motion where $0 \leq s \leq \infty$.

In fact:

i. $B_0 \stackrel{?}{=} 0$

$W_{t+0} - W_t \stackrel{?}{=} 0 \Longrightarrow W_t - W_t = 0$

ii. $B_R - B_S \stackrel{?}{\backsim} N(0, (R - S))$

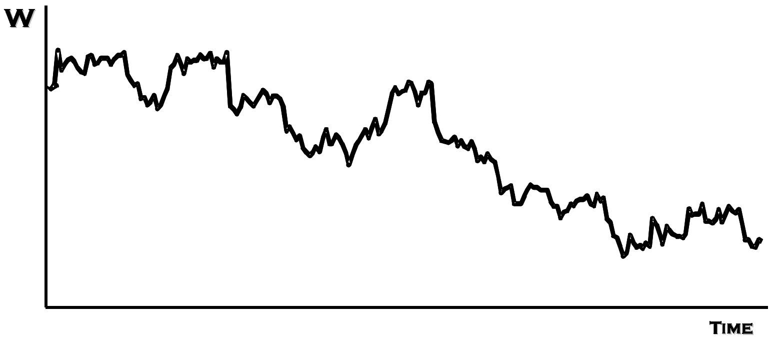

Figure 18:

$B_R - B_S = (W_{t+R} - W_t - W_{t+S} - W_t) = (W_{t+R} - W_{t+S})$
since any difference of random variables normally distributed is still normally distributed, so:
$E(W_{t+R}) = E(W_{t+S}) \Longrightarrow E(W_{t+R} - W_{t+S}) = 0$
$Var(W_{t+R} - W_{t+S}) = (R+t) - (t+S) = R - S$
$\therefore (W_{t+R} - W_{t+S}) \backsim N(0, (R-S))$

iii $(B_{t_2} - B_{t_1})$ *is $\overset{?}{\textit{independent}}$* from $(B_{t_4} - B_{t_3})$
where intervals $[t_4 - t_3)$ e $[t_2 - t_1)$ do not superimpose;
$(W_{t+t_2} - W_t - W_{t+t_1} - W_t)$ *is $\overset{?}{\textit{independent}}$* from
$(W_{t+t_3} - W_t - W_{t+t_2} - W_t) \Longrightarrow$
$\Longrightarrow (W_{t+t_2} - W_{t+t_1})$ *is $\overset{?}{\textit{independent}}$* from $(W_{t+t_3} - W_{t+t_2})$
since these intervals for hypothesis do not superimpose.

iv. B_S is continuous in t, since the difference of two random variables is still a continous random variable.
q.e.d..

For $s \to 0$ $\Delta W_t \to dW_t$ so it is possible to define in the continuum:
$dW_t \triangleq \varepsilon\sqrt{dt}$ where $\varepsilon \sim N(0,1)$
and therefore for what is above written:
$dW_t \sim N(o, dt)$
dW_t is called Wiener stochastic process and can be represented as shown in figure 18.
Its most general version is:
$dS_t = adt + bdW_t$
where
$dS_t \backsim N(a, b\sqrt{dt})$
Defining $a = \mu S_t$ and $b = \sigma S_t$ we obtain the following stochastic differential equation, known in probability as geometric Brownian motion:
$dS_t = \mu S_t dt + \sigma S_t dW_t$

Probability distribution of logarithmic stock-returns

$\ln\frac{S_t}{S_{t-dt}} \overset{?}{\backsim} N\left(\left(\mu-\frac{\sigma^2}{2}\right)dt, \sigma\sqrt{dt}\right)$

by taking the natural logarithm of the solution [2] we obtain:

$\ln S_t = \ln S_s + \left(\mu-\frac{\sigma^2}{2}\right)(t-s)+\sigma(W_t-W_s)$

or $\ln S_t - \ln S_s = \left(\mu-\frac{\sigma^2}{2}\right)(t-s)+\sigma(W_t-W_s)$

In this equation we recognize the generalized Wiener process and for what stated above we can conclude that $\ln S_t - \ln S_s$, is normally distributed with parameters $a=\mu-\frac{\sigma^2}{2}$ and $b=\sigma$

$\ln S_t - \ln S_s \backsim N\left(\left(\mu-\frac{\sigma^2}{2}\right)(t-s), \sigma\sqrt{(t-s)}\right)$

or:

$\ln\frac{S_t}{S_s} \backsim N\left(\left(\mu-\frac{\sigma^2}{2}\right)(t-s), \sigma\sqrt{(t-s)}\right)$

and in the continuous time:

$\ln\frac{S_t}{S_{t-dt}} \backsim N\left(\left(\mu-\frac{\sigma^2}{2}\right)dt, \sigma\sqrt{dt}\right)$

q.e.d.

Normal probability distribution and stock-price oscillation band

$P\left(S_{t-s}e^{\min} \le S_t \le S_{t-s}e^{\max}\right) \overset{?}{=} P(-z_{\frac{\varkappa}{2}} \le Z \le z_{\frac{\varkappa}{2}}) = \varkappa$

where:

$Z \sim N(0,1)$ i.e. the standard normal random variable.

$max = \sigma z_{\frac{\varkappa}{2}}\sqrt{t} + \left(\mu-\frac{\sigma^2}{2}\right)t$

$min = \sigma\left(-z_{\frac{\varkappa}{2}}\right)\sqrt{t} + \left(\mu-\frac{\sigma^2}{2}\right)t$

by defining:

$V = \ln\frac{S_t}{S_{t-dt}}$

then by hypothesis aforementioned V is a normal random variable with parameters $\left(\mu-\frac{\sigma^2}{2}\right)dt$ and $\sigma\sqrt{dt}$. By standardizing we get:

$Z = \frac{V-\left(\mu-\frac{\sigma^2}{2}\right)dt}{\sigma\sqrt{dt}} \sim N(0,1)$

Hence:

$P(-z_{\frac{\varkappa}{2}} \le Z \le z_{\frac{\varkappa}{2}}) = \varkappa$

By replacing the definition of Z :

$P\left(-z_{\frac{\varkappa}{2}} \le \frac{V-\left(\mu-\frac{\sigma^2}{2}\right)dt}{\sigma\sqrt{dt}} \le z_{\frac{\varkappa}{2}}\right) = \varkappa$

Hence:

$P\left(-z_{\frac{\varkappa}{2}}\sigma\sqrt{dt} + \left(\mu-\frac{\sigma^2}{2}\right)dt \le V \le z_{\frac{\varkappa}{2}}\sigma\sqrt{dt} + \left(\mu-\frac{\sigma^2}{2}\right)dt\right) = \varkappa$

By replacing the definition of V :

$P\left(-z_{\frac{\varkappa}{2}}\sigma\sqrt{dt} + \left(\mu-\frac{\sigma^2}{2}\right)dt \le \ln\frac{S_t}{S_{t-dt}} \le z_{\frac{\varkappa}{2}}\sigma\sqrt{dt} + \left(\mu-\frac{\sigma^2}{2}\right)dt\right) = \varkappa$

By taking the exponential function inside the parenthesis we get:

$P\left(e^{-z_{x/2}\sigma\sqrt{dt}+\left(\mu-\frac{\sigma^2}{2}\right)dt} \le \frac{S_t}{S_{t-dt}} \le e^{z_{x/2}\sigma\sqrt{dt}+\left(\mu-\frac{\sigma^2}{2}\right)dt}\right) = \varkappa$
By moving S_{t-dt} we get:
$P\left(S_{t-dt}e^{-z_{x/2}\sigma\sqrt{dt}+\left(\mu-\frac{\sigma^2}{2}\right)dt} \le S_t \le S_{t-dt}e^{z_{x/2}\sigma\sqrt{dt}+\left(\mu-\frac{\sigma^2}{2}\right)dt}\right) = \varkappa$
By replacing the definition of min and max we get:
$P\left(S_{t-s}e^{\min} \le S_t \le S_{t-s}e^{\max}\right) = \varkappa$
q.e.d.

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